Determinant

Case I :- if A is invertible :- A^{-1} exist :- $|A| \neq 0$ & $X = A^{-1}B$ (matrix method)

Case II :- if $|A| = 0$ → A^{-1} doesn't exist.

Then we will find :- a) $adj(A) \cdot B = 0$ → in this case S/m of L.E. may be Consistent or in-consistent. as has either infinite no. of Sol^n or has no solution.

→ b) $adj(A) \cdot B \neq 0$ → then S/m of L.E. is inconsistent & has no Sol^n.

Ex :- solve :- $2x - y = 5$, $x + y = 4$

Sol^n :- let :- $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Now A^{-1} → $|A| = 2 - (-1) = 3 \neq 0$

A^{-1} → exist

Now $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ so $X = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5+4 \\ -5+8 \end{bmatrix}$

$\Rightarrow X = \frac{1}{3}\begin{bmatrix} 9 \\ 3 \end{bmatrix} = \begin{bmatrix} 9/3 \\ 3/3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow x = 3$, $y = 1$ ✓

Determinant

Ex:- If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

Find A^{-1} & by using A^{-1} solve the S/M of given L.E.

L.E. :- $\begin{cases} 2x - 3y + 5z = 11 \\ 3x + 2y - 4z = -5 \\ x + y - 2z = -3 \end{cases}$

Soln:- $A^{-1} \rightarrow \frac{1}{|A|} \cdot adj(A)$

Sol:- $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$

$|A| = -6 + 5 = -1$

$|A| = -1 \neq 0 \rightarrow A^{-1}$ exist:-

$\rightarrow adj(A) = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 9 & +23 & 13 \end{bmatrix}^1 = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

So $A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ ✓

by using S/M :- Let.

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Sol:- $X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ So :- $x = 1$, $y = 2$, $z = 3$ ✓

Determinant

Q. The cost of 4 kg onion, 3 kg wheat & 2 kg Rice is ₹ 60, cost of 2 kg onion, 4 kg wheat & 6 kg Rice is ₹ 90, & cost of 6 kg onion, 2 kg wheat & 3 kg Rice is ₹ 70, find cost of each item / kg by matrix method.

Sol^{n}:- let Cost of onion $\Rightarrow x$ Rs. / kg.

— " — wheat $\Rightarrow y$ Rs / kg

— " — Rice $\Rightarrow z$ Rs / kg

So:- according to given statement:-

$$\Rightarrow \begin{cases} 4x + 3y + 2z = 60 \\ 2x + 4y + 6z = 90 \\ 6x + 2y + 3z = 70 \end{cases} \rightarrow 20 + 24 + 16 = 60 \text{ A}$$

Let: $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$\rightarrow$

$A^{-1} \Rightarrow |A| = 4(12-12) - 3(6-36) + 2(4-24)$

$|A| = 90 - 40 = 50$

$$adj(A) = \begin{bmatrix} 0 & +30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

So: $X = A^{-1}B = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$$X = \frac{1}{50} \begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

So:- $x = 5$, $y = 8$, $z = 8$

So:- cost of onion = 5 Rs / kg.

— " wheat = 8 Rs / kg

= " Rice = 8 Rs / kg.