Determinant

Ex :- find inverse of $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Soln :- $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1[-3-0] = -3 \neq 0$

Here $|A| = -3 \neq 0$ → A is non-singular → A^{-1} exist

→ $adj(A) = \begin{bmatrix} -3 & +3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & +1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$ Ans

Ex :- for $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find a & b such that :-

$A^2 + aA + bI = 0$

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = 0$

$\Rightarrow 3a + b = -11$ $8 + 2a = 0$

$-12 + b = -11$ $a = -4$

$b = 1$ Ans

Determinant

Ex:- $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify:- $A^3 - 6A^2 + 9A - 4I = O$

& Find $A^{-1} = ?$

Solⁿ:- $A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

Now:- LHS → $A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

LHS = RHS

$\because A^3 - 6A^2 + 9A - 4I = O$

$\therefore A^3 \cdot A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4IA^{-1} = O \cdot A^{-1}$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = O$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I \Rightarrow A^{-1} = \frac{1}{4}[A^2 - 6A + 9I]$

$\Rightarrow A^{-1} = \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right)$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$

$\begin{bmatrix} - \\ - \end{bmatrix}_{2\times1}$ $A^{-1} = 2\times2$, $B = 2\times1$ → $\begin{bmatrix} - \\ - \end{bmatrix}_{2\times1}$

Determinant

Use of Determinant and Matrix:-

↓

for solving SIM of Linear equations and find Consistency / inconsistency of the SIM.

Consistency ↓ Sol^n exist

Inconsistency ↓ Sol^n exist $\not\exists$

For a SIM having 2 variables:-

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Let:- $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ (unknown), $B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

Now → if A is a non-singular Matrix:- A^{-1} → Exist

then:- $[AX = B]$ ✓

→ Multiply by A^{-1}.

⇒ $A^{-1}.AX = A^{-1}B$

⇒ $IX = A^{-1}B$

⇒ $\boxed{X = A^{-1}B}$

* SIM having 3 variables:-

→ $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Let:- $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

→ if $|A| \neq 0$ → A^{-1} exist:-

$AX = B$ ⇒ $\boxed{X = A^{-1}B}$