Determinant

$\Rightarrow$ $\underline{A} \rightarrow n$ order, $I \rightarrow$ same order, $\underline{B} \rightarrow$ exist such that.

$\rightarrow [AB = BA = I]$ —— (1)

$\Rightarrow$ Now:- $|A \cdot B| = |A| \cdot |B|$

$\rightarrow$ $\underline{A} \rightarrow$ inverse define $\rightarrow A^{-1} \rightarrow A \rightarrow$ non-singular matrix.

$\rightarrow \boxed{|A| \neq 0}$

$\Rightarrow$ Now:- $\underline{A}B = I \rightarrow |AB| = |I|$

$\Rightarrow [|A| \cdot |B| = 1] \rightarrow$ i.e. $\rightarrow \boxed{|A| \neq 0} \rightarrow$ means A is non-singular matrix.

Now:- $\rightarrow A \cdot adj(A) = adj(A) \cdot A = |A| \cdot I$

$\Rightarrow \left\{ A \cdot \underbrace{\left[\frac{1}{|A|} adj(A)\right]}_{B} = \underbrace{\left[\frac{1}{|A|} adj(A)\right]}_{B} A = I \right\}$ —— (2)

Here from eqn (1) & (2):-

$\left(A^{-1} = B = \left[\frac{1}{|A|} adj(A)\right]\right)$

Determinant

Ex:- $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify:- $\boxed{\text{adj}(A) \cdot A = |A| \cdot I}$ and also find $\rightarrow A^{-1} = ?$

$\Rightarrow \text{adj}(A) = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}' = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now $\text{adj}(A) \cdot A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$\Rightarrow \text{adj}(A) \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Now $|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1[16-9] - 3[4-3] + 3[3-4]$

$= 7 - 3 - 3 = 1$

Here $\boxed{|A| = 1}$ i.e. $\text{adj}(A) \cdot A = 1 \cdot I$

$\rightarrow \boxed{\text{adj}(A) \cdot A = I}$ H.P.

Now $A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$

$A^{-1} = \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$\boxed{A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}$ Ans

Determinant

Ex:- $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ $B \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

verify → $(AB)^{-1} = B^{-1}A^{-1}$ ✓

Solⁿ:- $AB = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$

Here AB^{-1} is define → $|AB| \neq 0$

$|AB| = \begin{vmatrix} -1 & 5 \\ 5 & -14 \end{vmatrix} = 14 - 25 = -11$

$\boxed{|AB| = -11 \neq 0}$ so AB^{-1} define

$\therefore (AB)^{-1} = \frac{1}{|AB|} \cdot adj(AB) = \frac{1}{-11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} =$

$(AB^{-1}) = \begin{bmatrix} 14/11 & +5/11 \\ 5/11 & 1/11 \end{bmatrix} = LHS$

RHS:- $B^{-1} \to B^{-1}$ define → $|B| \neq 0$

& $A^{-1} \to A^{-1}$ define → $|A| \neq 0$

So:- $|B| = \begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} = 3 - 2 = 1$, $|A| = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -8 - 3$

$\boxed{|B| = 1 \neq 0}$ $\boxed{|A| = -11 \neq 0}$

Now:- $B^{-1} = \frac{1}{|B|} \cdot adj(B)$, $A^{-1} = \frac{1}{|A|} \cdot adj(A)$

$B^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, $A^{-1} = \frac{1}{-11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

So $B^{-1}.A^{-1} = \frac{-1}{11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} = \begin{bmatrix} 14/11 & 5/11 \\ 5/11 & 1/11 \end{bmatrix}$ = RHS

Here: LHS = RHS. (H.P.)

Determinant

Ex:- for $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ Show that:- $\left[A^2-4A+I=0\right]$ & find $\left[A^{-1}=?\right]$

Sol^n:- $A^2=A\cdot A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$$A^2=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}$$

So Now: $LHS=A^2-4A+I$

$$=\begin{bmatrix}7 & 12\\4 & 7\end{bmatrix}-4\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=O=\text{RHS}\ \text{H.P.}$$

$\Rightarrow$ given: $A^2-4A+I=0$

$A^{-1}\rightarrow$ $A^2\cdot A^{-1}-4A\cdot A^{-1}+I\cdot A^{-1}=0\cdot A^{-1}$

$\Rightarrow$ $A-4I+A^{-1}=0$

$\Rightarrow$ $\boxed{A^{-1}=+4I-A}$

$\rightarrow A^{-1}=\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}-\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$

$$\boxed{A^{-1}=\begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}}$$