Determinant

Theorem:- i) $A(\text{Adj}(A)) = \text{adj}(A)\cdot A = |A|\cdot I$

-40
52
-12

Ex:- $A = \begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & 4 \\ 2 & -1 & 3 \end{bmatrix}$ → verify:- $\text{adj}(A)\cdot A = |A|\cdot I$

→ $\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$

→ $A_{11} = 7$ $\quad A_{21} = -1$ $\quad A_{31} = 13$

→ $A_{12} = 8$ $\quad A_{22} = 13$ $\quad A_{32} = -4$

→ $A_{13} = -2$ $\quad A_{23} = 5$ $\quad A_{33} = 1$

$\text{adj}(A) = \begin{bmatrix} 7 & 8 & -2 \\ -1 & 13 & 5 \\ 13 & -4 & 1 \end{bmatrix}' = \begin{bmatrix} 7 & -1 & 13 \\ 8 & 13 & -4 \\ -2 & 5 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & -1 & 13 \\ 8 & 13 & -4 \\ -2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & 4 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 33 & 0 & 0 \\ 0 & 33 & 0 \\ 0 & 0 & 33 \end{bmatrix} = 33 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \boxed{\text{adj}(A)\cdot A = 33\cdot I}$

So:- $|A| = \begin{vmatrix} 1 & 2 & -5 \\ 0 & 1 & 4 \\ 2 & -1 & 3 \end{vmatrix} = 1(7) - 0 + 2(8+5)$

$= 7 + 26 = 33$

So: $\text{adj}(A)\, A = |A|\cdot I$ H.P

Determinant

Theorem :- ii) a sq. Matrix is said to be <u>singular</u> if $\boxed{|A| = 0}$

Ex! $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \rightarrow |A| = 6 - 6 = 0$

$\boxed{|A| = 0} \rightarrow A \rightarrow$ singular matrix $\rightarrow A^{-1}$ not define.

iii) a sq. Matrix is said to be non-singular if $\boxed{|A| \neq 0}$

Ex!- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow |A| = 4 - 6 = -2$

$|A| = -2$

i.e. $\boxed{|A| \neq 0} \rightarrow \underline{A} \rightarrow$ non-singular: $\rightarrow A^{-1}$ exist.

iv) if A & B both are non-singular Matrix of same order. then AB & BA is also non singular Matrix of same order.

ABLES KOTA

Determinant

$\boxed{A \to n \times n \to |\text{adj}(A)| = |A|^{n-1}}$

(vi) $\boxed{\text{adj}(A) \cdot A = |A| \cdot I}$

$\to \text{adj}(A) \cdot A = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\to \text{adj}(A) \cdot A = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

Now take Determinant $\to$

$\Rightarrow |\text{adj}(A) \cdot A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$

$\Rightarrow |\text{adj}(A)| \cdot \cancel{|A|} = \cancel{|A|} \cdot |A| \cdot |A| \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow |\text{adj}(A)| = |A|^2 \cdot 1 \Rightarrow \boxed{|\text{adj}(A)| = |A|^{2}}$ ☆

Ex:- $A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} \to |\text{adj}(A)| = ?$

Solⁿ:- $\because$ A is sq. matrix with order 2×2

$\Rightarrow \therefore \boxed{|\text{adj}(A)| = |A|}$

$\Rightarrow$ LHS:- $\text{adj}(A) = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$

$|\text{adj}(A)| = \begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} = 12 + 2 = 14$

$\Rightarrow$ RHS $\Rightarrow \begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix} = 12 + 2 = 14$

- Here:- LHS = RHS

$\boxed{|\text{adj}(A)| = |A|}$ H.P.

(vii) if A is a matrix with order $n \times n$ then $|\text{adj}(A)| = |A|^{n-1}$