Determinant

adjoint :-

$A = \begin{bmatrix} 0 & 5 \\ -3 & 2 \end{bmatrix} \rightarrow adj(A) = \begin{bmatrix} 2 & -5 \\ 3 & 0 \end{bmatrix}$

Ex:- $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 7 & 9 \\ -2 & 0 & 4 \end{bmatrix} \rightarrow$ adjoint:- $adj(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$

Ex:- $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \rightarrow adj(A) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$

$\rightarrow$ Co-factor $\rightarrow A_{11} = (-1)^{1+1} M_{11} = 1.4 = 4$

$\rightarrow A_{12} = (-1)^{1+2} M_{12} = -(-3) = 3$

$\rightarrow A_{21} = (-1)^{1+2} M_{21} = -(2) = -2$

$\rightarrow A_{22} = (-1)^{2+2} M_{22} = +1 = 1$

$adj(A) = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

$\Rightarrow adj(A) = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}' = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$ ✓

directly

☆ $\Rightarrow A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow adj(A) = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

sign change

Interchange

Determinant

adjoint :- Theorem :- i) $\left[A \cdot \operatorname{adj}(A) = \operatorname{adj}(A) \cdot A = |A| \cdot I \right]$

prove: $\det A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ → adjoint :- $\operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$

$\rightarrow \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$$A \cdot \operatorname{adj}(A) = \begin{bmatrix} a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} & a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} & a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} \\ 0 & |A| & 0 \\ 0 & & |A| \end{bmatrix}$$

$$A \cdot \operatorname{adj}(A) = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix} = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| \cdot I$$

$$\Rightarrow \left[A \cdot \operatorname{adj}(A) = |A| \cdot I \right]$$