Determinant

Ex:- $A = \begin{vmatrix} 3 & 1 & 2 \\ 0 & 5 & -7 \\ 3 & 1 & 4 \end{vmatrix}$

Δ = Sum of product of Element of any row (Column) with their Corresponding Co-factor will give the value of determinant.

Co-factors :-

$\Rightarrow$ Co-factor of a_{11} = $A_{11} = (-1)^{1+1} M_{11} = (-1)^2 \begin{vmatrix} 5 & -7 \\ 1 & 4 \end{vmatrix} = 1(20+7) = 27$

$\rightarrow$ Cofactor of $a_{12} = A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 0 & -7 \\ 3 & 4 \end{vmatrix} = +21$

$\rightarrow$ —— ,, —— $a_{13} = A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 0 & 5 \\ 3 & 1 \end{vmatrix} = -15$

☆ Ex:- $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \rightarrow (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= \boxed{a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}}$ ✓

Determinant

$\Delta = a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$ ✓

Ex:- find Minor & Co-factor of each element

if $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ $\leftarrow R_3$ & verify

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$

& also find value of $\Delta = ?$

Soln:-

$M_{11} = -20$, $A_{11} = -20$

$M_{12} = -46$, $A_{12} = -(-46) = 46$

$M_{13} = 30$, $A_{13} = 30$

$M_{21} = -4$, $A_{21} = -(-4) = 4$

$M_{22} = -19$, $A_{22} = -19$

$M_{23} = 13$, $A_{23} = -13$

$M_{31} = -12$, $A_{31} = -12$

$M_{32} = -22$, $A_{32} = -(-22) = 22$

$M_{33} = 18$, $A_{33} = 18$

Now:-

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$

$\Rightarrow 2(-12) + (-3)(22) + 5(18) = 0$

$\rightarrow -24 - 66 + 90 = 0$

$\rightarrow -90 + 90 = 0$ H.P.

Now $\Delta = ?$

$\Delta = a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ (R_3)

$\Delta = 1(-12) + 5(22) + (-7)(18)$

$\Delta = -12 + 110 - 126 = -28 \Rightarrow \boxed{\Delta = -28}$ Ans

OR

C_2

$\Delta = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$

$= (-3)(46) + 0(-19) + 5(22)$

$= -138 + 110 = -28$

Determinant

1 Adjoint & Inverse of a Matrix :-

→ for a square matrix $A = [a_{ij}]_{m\times m}$ → adjoint of Matrix A is given as :- $adj(A) = [A_{ij}]^{T}_{m\times m}$

where A_{ij} is co-factor of element of Matrix.

Ex :- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ → $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ → $adj(A) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^{T} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^{T} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = adj(A)$$

Determinant

Ex:- find Adj(A) if $A = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$

→ $A = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$

→ $A_{11} = 7$

→ $A_{12} = -(-4) = 4$

→ $A_{21} = -2$

→ $A_{22} = 1$

→ $adj(A) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}'$

$= \begin{bmatrix} 7 & 4 \\ -2 & 1 \end{bmatrix}'$

$adj(A) = \begin{bmatrix} 7 & -2 \\ 4 & 1 \end{bmatrix}$ Ans