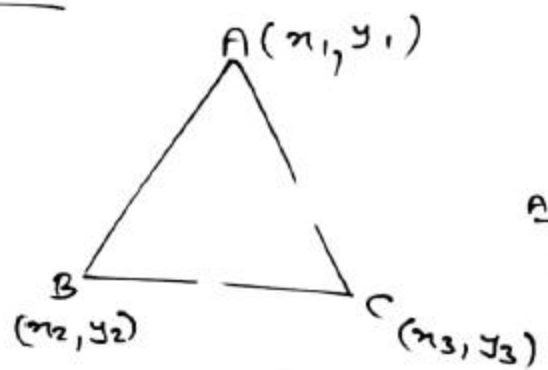
Determinant

Area of Triangle :-

if in a Δ all three vertices are (x_1, y_1) (x_2, y_2) & (x_3, y_3) are given then



A B C
$\Delta \to$ ↑
area = 0

$$\text{area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

i) area can never be -ive. that's why we consider +ive value.

ii) if area is given then we take both +ive & -ive value.

iii) if A, B & C are collinear then area of Δ is always zero

Determinant

Ex:- Find the area of Δ whose vert. are $(3,8)(-4,2)(5,1)$

Sol^{n}:- let $(x_1,y_1) \to (3,8)$
$(x_2,y_2) \to (-4,2)$
$(x_3,y_3) \to (5,1)$

Sol:- $\text{area}(\Delta) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_2 & 1 \end{vmatrix}$

$\text{area}(\Delta) = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$\text{ar}(\Delta) = \frac{1}{2}\left[3(2-1) - 8(-4-5) + 1(-4-10)\right]$

$= \frac{1}{2}\left[3 + 72 - 14\right] = \frac{1}{2}\left[61\right] = \frac{61}{2}$ sq. unit

Ex:-(2) Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

Sol^{n}:- $\because$ points are collinear i.e. $\text{area}(\Delta) = 0$

Sol:- $\text{area}(\Delta) = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} \to C_2 \to C_2 + C_1$

$\Rightarrow \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix} = \frac{1}{2}(a+b+c)\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}$ (C_2, C_3)

$= 0$ RHS = L.H.S

Ex:- if area of Δ is 35 sq. units with vertices $(2,-6)$ $(5,4)$ & $(k,4)$ then k is:-

Sol^{n}:- $\because \text{area}(\Delta) = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35$

$\Rightarrow \frac{1}{2}\left[2(0) + 6(5-k) + 1(20-4k)\right] = \pm 35$

$\Rightarrow 30 - 6k + 20 - 4k = \pm 70 \Rightarrow -10k + 50 = \pm 70 \Rightarrow \left[-10k = \pm 70 - 50\right]$

+ive $\to$ $-10k = +70 - 50$
$-10k = 20 \Rightarrow (k = -2)$

-ive $\to$ $+10k = -70 - 50 = +120$
$\Rightarrow k = \frac{120}{10} \Rightarrow (k = 12)$ $\to$ $(k = -2, 12)$ Ans

Determinant

Ex:- Find eq. of line joining (1, 2) & (3, 6) using determinants ?

Soln:-

→ Let a point (x, y) on line AB

So A, B & C are collinear therefore area of $\Delta ABC = 0$.

So:- $\text{area}(\Delta) = 0 = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ x & y & 1 \\ 3 & 6 & 1 \end{vmatrix}$

$\Rightarrow 0 = \frac{1}{2}\left[1(y-6) - 2(x-3) + 1(6x-3y)\right]$

$\Rightarrow y - 6 - 2x + 6 + 6x - 3y \Rightarrow -2y + 4x = 0$

$\Rightarrow 4x - 2y = 0 \Rightarrow 4x = 2y \Rightarrow 2x = y$ Ans