Determinant

Ex:- prove :-

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Sol^n:-

Ex:- Show that:-

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) = \underline{bc+ac+ab+abc}$$

LHS:-

$$\Rightarrow abc\begin{vmatrix} \frac{1}{a}+1 & 1/a & 1/a \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix} \Rightarrow R_1 \to R_1+R_2+R_3$$

$$\Rightarrow abc\begin{vmatrix} 1/a+1+1/b+1/c & 1/a+1/b+1+1/c & 1/a+1/b+1/c+1 \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix}$$

$$\Rightarrow abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)\begin{vmatrix} 1 & 1 & 1 \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix} \to C_1 \to C_1 - C_2$$

$$\Rightarrow abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)\begin{vmatrix} 0 & 1 & 1 \\ -1 & 1/b+1 & 1/b \\ 0 & 1/c & 1/c+1 \end{vmatrix}$$

$$\Rightarrow abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)\left[-(-1)\left\{\cancel{1/c}+1-\cancel{1/c}\right\}\right]$$

$$\Rightarrow \text{—}\,''\,\text{—}\ \left[1(1)\right]$$

$$\Rightarrow abc\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right] = \text{RHS} \quad \underline{\text{H.P}}$$

Determinant

Ex:- prove

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

$$\boxed{C_1 \to C_1 + C_2 \times 9}$$

Sol^n:-

$$\begin{vmatrix} 2+7\times 9 & 7 & 65 \\ 3+8\times 9 & 8 & 75 \\ 5+9\times 9 & 9 & 86 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 65 & 7 & 65 \\ 75 & 8 & 75 \\ 86 & 9 & 86 \end{vmatrix} = 0$$

Ex:-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \underline{(a-b)}\,(\underline{b-c})(c-a)$$

Sol^n:- $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 1-1 & a-b & a^2-b^2 \\ 1-1 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & a-b & (a+b)(a-b) \\ 0 & b-c & (b+c)(b-c) \\ 1 & c & c^2 \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$(a-b)(b-c)\left[1\{b+c-(a+b)\}\right]$$

$$(a-b)(b-c)(\cancel{b}+c-a-\cancel{b})$$

$$\boxed{(a-b)(b-c)(c-a)} \to \text{RHS}$$

Determinant

Ex.:- $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

Soln:- $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow C_1 \to C_1 - C_2 \Rightarrow (5x+4)\begin{vmatrix} 0 & 1 & 1 \\ x-4 & x+4 & 2x \\ 0 & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow (5x+4)\left[-(x-4)\left[x+4-2x\right]\right]$$

$$\Rightarrow (5x+4)\left[-(x-4)(-x+4)\right]$$

$$\Rightarrow (5x+4)\left[(-x+4)(-x+4)\right]$$

$$\Rightarrow \boxed{(5x+4)(4-x)^2}$$

RHS

L.H.S.

Determinant

Ex! - $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

LHS: -

→ a, b, c → common! - from R_1, R_2 & R_3 respectively

$\Rightarrow abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

→ a, b, c common from c_1, c_2 & c_3 res.

$\Rightarrow abc(abc) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\Rightarrow a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$C_1 \to C_1 + C_2$

$\to a^2b^2c^2 \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$

$\to a^2b^2c^2 \left[2(1+1)\right]$

$\Rightarrow 4a^2b^2c^2 \to$ RHS

H.P

Determinant

Ex:- $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

Sol^n:- $\rightarrow$ Row $\longleftrightarrow$ column

$$\Delta = \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$$\Delta = (-)(-)(-)\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$\Delta = -\Delta$$

$\Rightarrow \Delta + \Delta = 0 \rightarrow 2\Delta = 0 \Rightarrow \Delta = 0$

H.P