Determinant

vi) $|A| = \begin{vmatrix} x+\lambda_1 & y+\lambda_2 & z+\lambda_3 \\ a & b & c \\ x & y & z \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ a & b & c \\ x & y & z \end{vmatrix}$

RHS

LHS:-

$(x+\lambda_1)[bz-cy] - (y+\lambda_2)[az-cx] + (z+\lambda_3)[ay-bx]$

$= x(bz-cy) + \lambda_1(bz-cy) - y(az-cx) - \lambda_2(az-cx) + z(ay-bx) + \lambda_3(ay-bx)$

LHS → $x(bz-cy) - y(az-cx) + z(ay-bx) + \lambda_1(bz-cy) - \lambda_2(az-cx) + \lambda_3(ay-bx)$

RHS :- $\underbrace{x(bz-cy) - y(az-cx) + z(ay-bx)} + \lambda_1(bz-cy) - \lambda_2(az-cx) + \lambda_3(ay-bx)$

Prove

Ex:- $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$

pro → 6

LHS

$\begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$

prot-3 prot-5

$0 + 0$

$= 0 =$ RHS H.P.

Determinant

viii) $|A| = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \Rightarrow R_1 \to R_1 + kR_2 \Rightarrow |B| = \begin{vmatrix} a+kx & b+ky & c+kz \\ x & y & z \\ p & q & r \end{vmatrix}$

prove $\boxed{|A| = |B|}$

$$|B| = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} + \begin{vmatrix} kx & ky & kz \\ x & y & z \\ p & q & r \end{vmatrix}$$

$$|B| = |A| + 0$$

$$\boxed{|B| = |A|}$$

Ex:- $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

Sol^n:- $R_2 \to R_2 - 2R_1$ $\quad R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a-2a & a+0 & 2a+b+0 \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} \Rightarrow \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\to \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix} = a[a^2 - 0] = a^3 = \text{RHS}$$ h.p.

Determinant

Ex:- Prove $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

$\Rightarrow R_1 \to R_1 - (R_2 + R_3)$

$$\begin{vmatrix} b+c-(b+c) & a-(c+a+c) & a-(b+a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \Rightarrow -(-2c)\left[(a+b)b - bc\right] - 2b\left[bc - c(a+c)\right]$$

$$= 2c\left[ab + b^2 - bc\right] - 2b\left[bc - ac - c^2\right]$$

$$\Rightarrow 2abc + 2b^2c - \cancel{2bc^2} - 2b^2c + 2abc + \cancel{2bc^2}$$

$$\Rightarrow 4abc \quad \text{RHS} \quad \text{H.P}$$