

# Determinant #

Ex: ①  $|A| = \begin{vmatrix} 2 & 4 \\ 4 & 10 \end{vmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix}$

if  $A = \begin{bmatrix} 2 & 4 \\ 4 & 10 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

$A = 2B$

$A = 3B$

$|A| = 3^2 |B|$

$|A| = 9|B|$

$\Rightarrow |A| = 20 - 16$

$|A| = 4$

$|B| = 5 - 4$

$|B| = 1$

Hence:-

$|A| = 4 \cdot |B|$

$|A| = 2^{\text{order}} \cdot |B| \Rightarrow |A| = k^n |B|$

Concl.  $\rightarrow A = kB \rightarrow |A| = k^n |B|$

Where  $k$  is scalar by which matrix  $B$  is multiply.

$n \rightarrow$  order of matrix / Determinant

Ex ②  $|A| = \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 5 & 3 \end{vmatrix}$ ,  $|B| = \begin{vmatrix} 2 & 0 & 2 \\ 4 & -2 & 0 \\ 2 & 10 & 6 \end{vmatrix}$

$|A| = 1[-3-0] + 1(10+1) = -3+11 = 8 \Rightarrow |A| = 8$

$|B| = 2(-12-0) + 2(4+4) = -24+88 = 64$

$|B| = 64$

$|A| = 8$

$|B| = 8|A|$

$|B| = 2^{\text{order}} |A| \Rightarrow |B| = k^n |A|$

$2A = B$

$|B| = 2^3 |A|$

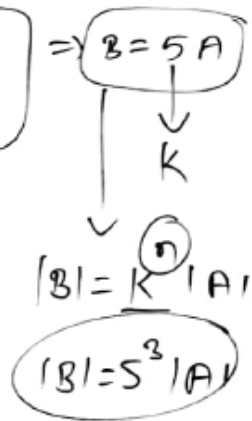
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Ex:-  $\checkmark$   $A = \begin{bmatrix} 1 & 0 & 7 \\ -1 & 3 & 5 \\ 4 & 0 & 2 \end{bmatrix} \rightarrow B = \begin{bmatrix} 5 & 0 & 35 \\ -5 & 15 & 25 \\ 20 & 0 & 10 \end{bmatrix} \Rightarrow B = 5 \begin{bmatrix} 1 & 0 & 7 \\ -1 & 3 & 5 \\ 4 & 0 & 2 \end{bmatrix} \Rightarrow B = 5A$

Find  $|A|$  &  $|B| = ?$

Sol<sup>n</sup>:-  $|A| = -0 + 3(2 - 28) - 0$   
 $|A| = 3x - 26 = -78$   
 $|A| = -78 \checkmark$

$|B| = 15(50 - 700)$   
 $= 15x - 650$   
 $|B| = -9750$



Here:-  $B = 5A$

then:-  $|B| = 5^3 |A|$

$|B| = 125 |A|$

$|B| = 125x - 78$

$|B| = -9750$

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i)

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & 3 \\ 4 & 3 & 2 \end{vmatrix} \xrightarrow{R_i \leftrightarrow C_i} |B| = \begin{vmatrix} 1 & 2 & 4 \\ 0 & 5 & 3 \\ -1 & 3 & 2 \end{vmatrix} \rightarrow |B| = 5(2+4) - 3(3+2) = 30 - 15 = \underline{15}$$

$$|A| = 1(10-9) - 1(6-20)$$

$$\underline{|A|} = 1 + 14 = \underline{15}$$

$$|B| = 1(10-9) - 1(6-20)$$

$$\underline{|B|} = 1 + 14 = \underline{15}$$

ii)

$$|A| = \begin{vmatrix} 3 & 1 & 2 \\ -7 & 1 & 0 \\ 5 & 3 & 1 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_3} |B| = \begin{vmatrix} 5 & 3 & 1 \\ -7 & 1 & 0 \\ 3 & 1 & 2 \end{vmatrix}$$

$$|A| = 2(-21-5) + 1(3+7)$$

$$|A| = -52 + 10 = \underline{-42}$$

$$|B| = 1(-7-3) + 2(5+21)$$

$$= -10 + 52$$

$$|B| = \underline{42}$$

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iii)  $|A| = \begin{vmatrix} 1 & -3 & 7 \\ 0 & 1 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$

$$|A| = 1(7-7) - 2(-3+3) = 0 - 0$$

$|A| = 0$

$$\begin{aligned} \Delta &= -\Delta \\ \Delta + \Delta &= 0 \\ 2\Delta &= 0 \\ \Delta &= 0 \end{aligned}$$

v)  $|A| = \begin{vmatrix} 1 & 2 & 3 \\ -5 & 0 & 7 \\ 2 & 4 & 6 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & 3 \\ -5 & 0 & 7 \\ 1 & 2 & 3 \end{vmatrix} = 2(0) = 0$

$$|A| = -2(-30-14) - 4(7+15)$$

$$|A| = 88 - 88 = 0$$

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{1}{2} \quad \frac{3}{6} \quad \frac{1}{2}$$

iv)  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow 5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$

$$5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \Rightarrow 5|A| = \begin{vmatrix} 5 & 10 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 15 & 20 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 15 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 10 \\ 3 & 20 \end{vmatrix}$$

$$|A| = 4 - 6 = -2 \rightarrow 5|A| = -10$$

$$5|A| = 5 \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$