

Ques:- $Ax = B$, $B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -1/2 & -1/2 \\ -4 & 3/4 & 5/4 \\ 2 & -1/4 & -3/4 \end{bmatrix}$

then $x = ?$

A) $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

B) $\begin{bmatrix} -1/2 \\ -1/2 \\ 2 \end{bmatrix}$

C) $\begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$

D) $\begin{bmatrix} 3 \\ 3/4 \\ -3/4 \end{bmatrix}$

H Matrix :- $x = A^{-1}B$

Solⁿ:- $Ax = B \rightarrow x = \frac{B}{A} = \frac{1}{A} \cdot B = A^{-1}B$

\Rightarrow Multiply both side by A^{-1}

$\Rightarrow A^{-1}Ax = A^{-1}B$

$\Rightarrow Ix = A^{-1}B$

$\Rightarrow x = A^{-1}B$

So $x = \begin{bmatrix} 3 & -1/2 & -1/2 \\ -4 & 3/4 & 5/4 \\ 2 & -1/4 & -3/4 \end{bmatrix} \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} = \begin{bmatrix} 27 - 1/2 \times 52 + 0 \\ -36 + 3/4 \times 52 + 0 \\ 18 - 1/4 \times 52 + 0 \end{bmatrix} = \begin{bmatrix} 27 - 26 \\ -36 + 39 \\ 18 - 13 \end{bmatrix}$

$x = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ ✓

2) A
 \downarrow
 $2 \cdot 2^{-1} A$
 $\frac{2}{2} A$
 A

H Matrix:

Q.1) $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$

if $AA^T = I_2$ find $p = ?$

A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{\sqrt{6}}$

C) $\frac{1}{\sqrt{3}}$ D) 0

Soln:- $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$

$\Rightarrow AA^T = I_3 \Rightarrow \begin{bmatrix} 0+4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2 \\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2 \\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$q_{12} \Rightarrow 2q^2 - r^2 = 0 \Rightarrow 2q^2 = r^2 \quad \text{--- (1)}$

$q_{22} \Rightarrow p^2 + q^2 + r^2 = 1 \Rightarrow p^2 + q^2 + 2q^2 = 1 \Rightarrow p^2 + 3q^2 = 1 \quad \text{--- (2)}$

Q.2) if A, B & C are sq. matrix of same order such that:- $AB=0$ & $BC=I$ then

$\rightarrow (A+B)^2(A+C)^2 = ?$

- A) 0 B) $2I$ C) I D) $3I$

$q_{11} = 4q^2 + r^2 = 1$

$\Rightarrow 4q^2 + 2q^2 = 1$ --- from eq (1)

$\Rightarrow 6q^2 = 1 \Rightarrow q^2 = \frac{1}{6}$ --- (3)

put q^2 from eq (3) in eq (2):-

$\Rightarrow p^2 + 3 \cdot \frac{1}{6} = 1 \Rightarrow p^2 + \frac{1}{2} = 1$

$\Rightarrow p^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$p = \frac{1}{\sqrt{2}}$ ✓

Matrix:

Q.31 If A & B are sq. matrix of order 3×3 which satisfy $AB = A$ & $BA = B$ then $(A+B)^6 = ?$

- A) $6(A+B)$ B) $6I$ C) $32(A+B)$ D) $64I$

$a \cdot b = 0$
 $a = 0$
 or
 $b = 0$

$AB = 0$
 \downarrow
 it doesn't mean
 $\checkmark A=0$ or $\checkmark B=0$

$B = B^{-1}$
 \downarrow
 null matrix

$AB = I$
 \downarrow
 $B = A^{-1}$
 \downarrow
 $A \cdot A^{-1} = I$

Q.2) If A, B & C are sq. matrix of same order such that:- $AB = 0$ & $BC = I$ then

- $\rightarrow (A+B)^2 (A+C)^2 = ?$
 A) 0 B) $2I$ C) I D) $3I$

Soln:- $AB = 0$ (1) & $BC = I$ (2)
 From eqⁿ (1):-

$\checkmark A = 0$ or $B = 0$

$BC = I$ (2)
 $\downarrow \downarrow$
 $B B^{-1} = I$
 i.e. $\Rightarrow C = B^{-1}$ or $B = C^{-1}$
 Here B^{-1} is define
 it means:- $B = 0$ not possible

then here $A = 0$

So:- $(A+B)^2 (A+C)^2 = (0+B)^2 (0+C)^2$
 $= (B)^2 (C)^2 = (BC)^2 = I^2 = I = I$

Matrix:

Q.3) If A & B are sq. matrix of order 3x3
 which satisfy $AB=A$ & $BA=B$
 then $(A+B)^6 = ? \rightarrow (A+B)^4 = ?$
 A) $6(A+B)$ B) $6I$ C) $32(A+B)$ D) $64I$

Q. $A = \begin{vmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{vmatrix}$ $B = \begin{vmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{vmatrix}$
 $AB = ?$ A) A B) B C) 0 D) I

Solⁿ:-

$$AB = A$$

Multi. by A^{-1}

$$\Rightarrow A^{-1} \cdot AB = A^{-1} \cdot A$$

$$\Rightarrow IB = I$$

$$\Rightarrow B = I$$

&

$$BA = B$$

Multi. by B^{-1}

$$B^{-1} \cdot BA = B^{-1} \cdot B$$

$$IA = I$$

$$A = I$$

$$\text{So: } (A+B)^6 = (I+I)^6 = (2I)^6 = 2^6 \cdot I^6 = 64I$$

$$(I+I)^4 = (2I)^4 = 16I$$