Matrix

$CI = IC = C$

✓ $[AI = IA = A]$

✓ $B(AC) = (BA)C$

$[AB = BA = I]$

$A \to$ inverse of $B \Rightarrow A^{-1} = B$

✓ $B \to$ inverse of $A = A = B^{-1}$

✓ $[AA^{-1} = B \cdot B^{-1} = I]$

$\underline{A} \to$ inverse of $A \to B$ (unique) $\Rightarrow AB = \underline{BA} = I$

$A \to$ inverse of $A \to C$ (unique) $\Rightarrow AC = CA = I$

if inverse of Matrix A is unique :- it means $\to$ $\boxed{B = C}$

Now:- $[B = BI = \underline{B(AC)} = (BA)C = IC = C] \Rightarrow B = C$

[$AI = IA = A$]

Matrix

$2 \to \cancel{2} \times 2^{-1}$ 1

$2 \times 2^{-1} \to 2^{-1} \times 2$

if A & B are square invertible Matrix of same order:-

$\Rightarrow \boxed{(AB)^{-1} = B^{-1} A^{-1}}$

$AI = IA = A$

$\because$ $A \cdot A^{-1} = I$

$\therefore \Rightarrow (AB)(AB)^{-1} = I$

$\Rightarrow$ Multiply both side:- by A^{-1}

$\Rightarrow A^{-1} \cdot AB(AB)^{-1} = A^{-1} \cdot I$

$\Rightarrow IB(AB)^{-1} = A^{-1}$

$\Rightarrow B(AB)^{-1} = A^{-1}$

$\to$ Multiply both sides by B^{-1}:-

$\Rightarrow B^{-1} \cdot B(AB)^{-1} = B^{-1} \cdot A^{-1}$

$I(AB)^{-1} = B^{-1} \cdot A^{-1}$

$\Rightarrow \boxed{(AB)^{-1} = B^{-1} \cdot A^{-1}}$

Matrix # $\boxed{A = AI}$ $\left\{ AI = \boxed{IA = A} \right.$

How to find inverse of a Matrix:- Column $\boxed{A = IA}$ → Row

⇒ for a Matrix A → if we find inverse of matrix A then simply right :- $[A = IA]$ → for $\boxed{\text{row}}$ Elementary operation:-

↓ Convert such as:-

$[I = BA]$ → Here B is inverse of A.

i.e. $\boxed{B = A^{-1}}$

Now for Column Elementary operation:- $[A = AI]$

↓ Convert such as:-

$[I = AB]$ → i.e. B inverse of A.

or $\boxed{A^{-1} = B}$

Matrix

Ex:- if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, find A^{-1} if exists.

Sol^n:- i) we will find A^{-1} by using Row Elementary operation.

$\therefore$ $\boxed{A = IA} \rightarrow I = BA$

So!
$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2-2 & -1-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0-2 & 1-0 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$R_2 \rightarrow \frac{-1}{5} R_2$
$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix} A$$

$\Rightarrow R_1 \rightarrow R_1 - 2R_2$

$$\Rightarrow \begin{bmatrix} 1-0 & 2-2(1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-2(2/5) & 0-2(-1/5) \\ 2/5 & -1/5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} A$$

i.e. $\Rightarrow$ $\boxed{I = BA}$ $\rightarrow$ i.e. B is inverse of A

So $A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$ Ans.