Matrix

$[(A+B)' = A' + B' \,/\, (A')' = A$

✲ if A is a square Matrix with all real no. than:-

$(A + A')$ is a Symmetric Matrix.

Ex:- $A = \begin{bmatrix} 4 & 1 & 7 \\ 1 & 3 & 8 \\ 7 & 8 & 5 \end{bmatrix} \rightarrow A' = \begin{bmatrix} 4 & 1 & 7 \\ 1 & 3 & 8 \\ 7 & 8 & 5 \end{bmatrix}$

$B' = A' + (A')'$

$B' = A' + A = B$

$\boxed{B' = B}$

Now $A + A' = \begin{bmatrix} 8 & 2 & 14 \\ 2 & 6 & 16 \\ 14 & 16 & 10 \end{bmatrix} = B$

then:- $B' = (A+A)' = \begin{bmatrix} 8 & 2 & 14 \\ 2 & 6 & 16 \\ 14 & 16 & 10 \end{bmatrix} = B$

$A + A' \rightarrow$ symm.

let:- $B = A + A' \rightarrow$ symm.

$\Rightarrow \boxed{B = B'}$

$B' = (A + A')'$

Matrix

✱ if A is a square Matrix with all real no. then:-

$\underline{(A - A')}$ is a skew Symmetric Matrix.

$\Rightarrow$ $\underline{(A - A')} \rightarrow$ skew symmetric Matrix.

$\Rightarrow$ $C = (A - A') \rightarrow \boxed{C' = -C}$

$\downarrow$

transpose:- $C' = (A - A')'$

$\Rightarrow$ $C' = A' - (A')'$

$\Rightarrow$ $C' = \underline{A' - A} = -(A - A')$

$\Rightarrow$ $C' = -(A - A') = -C \Rightarrow C' = -C$

Matrix

iii) for a square Matrix A with all real no. we can express Matrix A as the sum of symm. & skew symm. Matrix.

$$\Rightarrow \boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$$

A = Symm + skew symm. Ma.

$$\boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$$

$$\frac{1}{2}(A+\cancel{A'}) + \frac{1}{2}(A-\cancel{A'})$$

$$\cancel{2}(A)$$

Matrix

Ex:- Express → $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symm. & skew symm. Matrix.

Sol^n:- $\boxed{A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')}$

$\Rightarrow$ Now $A + A' = B$, $A - A' = C$

$\rightarrow$ $\boxed{A = \frac{1}{2}B + \frac{1}{2}C}$

So:- $A' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} \Rightarrow A + A' = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = B$, $B' = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} \Rightarrow \boxed{B' = B}$

Now $A - A' = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = C \Rightarrow C' = \begin{bmatrix} 0 & 1 & 5 \\ -1 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = -C \Rightarrow \boxed{C' = -C}$

So: $A = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$

Matrix

Ex:- express → $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symm. & skew symm. Matrix.

Sol: $A = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$

A

$A = \begin{bmatrix} 2 & \frac{-3}{2} - \frac{1}{2} & \frac{-3}{2} - \frac{5}{2} \\ \frac{-3}{2} + \frac{1}{2} & 3 & 4 \\ \frac{-3}{2} + \frac{5}{2} & -2 & -3 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ H.P.

Matrix

Ques:- if $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Sol^n:- $A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow \quad A' \cdot A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A' \cdot A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha \cdot \sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$A' \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow I \Rightarrow A' \cdot A = I$