Matrix

Transpose of a Matrix :-

if we interchange row & column of a matrix then the new Matrix is known as Transpose of a Matrix.

Ex:- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2} \longrightarrow A' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3} = A^T$

$\left\{ A = [a_{ij}]_{mn} \qquad A' = [a_{ji}]_{nm} \right\}$

Ex:- $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -7 & 1 \\ 2 & 3 & 0 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -7 & 3 \\ 5 & 1 & 0 \end{bmatrix}$ ✓

i) $A = \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} \Rightarrow (A')' = \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix} = A$

Properties of Transpose of a Matrix :-

i) $(A')' = A$

ii) $KA' = (KA)'$

Matrix

ii) Ex:- $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -7 \end{bmatrix} \rightarrow KA' = (KA)'$

$\rightarrow$ LHS $= KA' = K\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} K & 0 \\ 2K & K \\ 5K & -7K \end{bmatrix}$

$\rightarrow$ RHS $\Rightarrow (\underline{KA})' = \begin{bmatrix} K & 2K & 5K \\ 0 & K & -7K \end{bmatrix}' = \begin{bmatrix} K & 0 \\ 2K & K \\ 5K & -7K \end{bmatrix}$

iii) Ex:- $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 7 \\ 1 & 2 & 0 \end{bmatrix}$

LHS $\rightarrow A+B = \begin{bmatrix} 4 & 9 & 14 \\ 3 & 2 & 4 \end{bmatrix} \Rightarrow (A+B)' = \begin{bmatrix} 4 & 3 \\ 9 & 2 \\ 14 & 4 \end{bmatrix}$

RHS $\rightarrow A' = \begin{bmatrix} 1 & 2 \\ 5 & 0 \\ 7 & 4 \end{bmatrix}$, $B' = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 7 & 0 \end{bmatrix} \Rightarrow A'+B' = \begin{bmatrix} 4 & 3 \\ 9 & 2 \\ 14 & 4 \end{bmatrix}$ ✓

Properties of Transpose of a Matrix:-

i) $(A')' = A$

ii) $KA' = (KA)'$

iii) $(A+B)' = A' + B'$

Matrix

(iv) Ex:- $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & 5 \\ 3 & -2 & 0 \end{bmatrix}_{3\times3}$

LHS → $(AB)' \Rightarrow AB = \begin{bmatrix} 1+0+12 & 4+2-8 & -2+10+0 \\ 1+0+6 & 4+3-4 & -2+15+0 \end{bmatrix} = \begin{bmatrix} 13 & -2 & 8 \\ 7 & 3 & 13 \end{bmatrix}$

So:- $(AB)' = \begin{bmatrix} 13 & 7 \\ -2 & 3 \\ 8 & 13 \end{bmatrix}$ ✓ = LHS

RHS:- ✓ $B'A' \Rightarrow$? $B' = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 1 & -2 \\ -2 & 5 & 0 \end{bmatrix}_{3\times3}$, $A' = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}_{3\times2}$

So:- $\underset{3\times2}{B'A'} = \begin{bmatrix} 1+0+12 & 1+0+6 \\ 4+2-8 & 4+3-4 \\ -2+10+0 & -2+15+0 \end{bmatrix} = \begin{bmatrix} 13 & 7 \\ -2 & 3 \\ 8 & 13 \end{bmatrix}$ = RHS

LHS = RHS

$(AB)' = B' \cdot A'$

Properties of Transpose of a Matrix:-

i) $(A')' = A$

ii) $KA' = (KA)'$

iii) $(A+B)' = A' + B'$

iv) $(AB)' = B' \cdot A'$

Matrix

Symmetric & Skew symmetric Matrix :-

→ if A is a square Matrix such that.

→ $[A' = A]$ → then A is called symm. Matrix.

$[a_{ij} = a_{ji}]$ ✓ → for all value of i & j.

↓

Symm. Matrix

Ex :-

$$A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 7 & 3 \\ -5 & 3 & 4 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 7 & 3 \\ -5 & 3 & 4 \end{bmatrix}$$

$a_{12} = a_{21}$

$a_{32} = a_{23}$

$a_{22} = a_{22}$

$a_{31} = a_{13}$

⇒ $\boxed{A' = A}$ → we can say A is Symmetric Matrix.

Matrix

Symmetric & Skew symmetric Matrix:-

✻ if for a square Matrix A :- $\left[A' = -A\right]$ → then matrix A is called skew symm. Matrix

Ex:- $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}$

→ Skew Symm. Matrix!

$a_{ij} = -a_{ji}$

→ $A' = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 4 \\ 2 & -4 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}$

→ $a_{21} = -a_{12} \Rightarrow -1 = -1$

→ $a_{32} = -a_{23} \Rightarrow 4 = -(-4) \Rightarrow 4 = 4$

⇒ $\boxed{A' = -A}$ → Hence proved

it mean A is skew Symm. Matrix.

✻ $\Rightarrow a_{22} = -a_{22}$

$\Rightarrow a_{22} + a_{22} = 0$

$= 2 \cdot a_{22} = 0$

$\Rightarrow a_{22} = 0$

$a_{11} = 0$

$a_{33} = 0$