Matrix

Ex:- $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 5 & 4 & 1 \end{bmatrix}_{3\times3}$, $B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 1 & 5 \end{bmatrix}_{3\times2}$, $C = \begin{bmatrix} 1 & -1 & 2 & 5 \\ 6 & 1 & 7 & 3 \end{bmatrix}_{2\times4}$

Find :- $(AB)C$ & $A(BC)$ & also show :- $(AB)C = A(BC)$

Sol^n:- $\Rightarrow \underset{3\times2}{AB} \Rightarrow \begin{bmatrix} 1+0+2 & 3+0+10 \\ -1+4+3 & -3+0+15 \\ 5+8+1 & 15+0+5 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 13 \\ 6 & 12 \\ 14 & 20 \end{bmatrix}_{3\times2}$; $C = \begin{bmatrix} 1 & -1 & 2 & 5 \\ 6 & 1 & 7 & 3 \end{bmatrix}_{2\times4}$

$\Rightarrow \underset{3\times4}{(AB)C} = \begin{bmatrix} 3+78 & -3+13 & 6+91 & 15+39 \\ 6+72 & -6+12 & 12+84 & 30+36 \\ 14+120 & -14+20 & 28+140 & 70+60 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 81 & 10 & 97 & 54 \\ 78 & 6 & 96 & 66 \\ 134 & 6 & 168 & 130 \end{bmatrix} \rightarrow$ LHS

Now: $\underset{3\times4}{BC} = \begin{bmatrix} 1+18 & -1+3 & 2+21 & 5+9 \\ 2+0 & -2+0 & 4+0 & 10+0 \\ 1+30 & -1+5 & 2+35 & 5+15 \end{bmatrix}$

$BC = \begin{bmatrix} 19 & 2 & 23 & 14 \\ 2 & -2 & 4 & 10 \\ 31 & 4 & 37 & 20 \end{bmatrix}$

$97 - 19 = 78$ $\quad$ $119 - 23 = 96$

Matrix

Ex:- $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 5 & 4 & 1 \end{bmatrix}_{3\times3}$, $B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 1 & 5 \end{bmatrix}_{3\times2}$, $C = \begin{bmatrix} 1 & -1 & 2 & 5 \\ 6 & 1 & 7 & 3 \end{bmatrix}_{2\times4}$

& also show:- $(AB)C = A(BC)$

Now:- $\underset{3\times4}{A(BC)} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 5 & 4 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} 19 & 2 & 23 & 14 \\ 2 & -2 & 4 & 10 \\ 31 & 4 & 37 & 20 \end{bmatrix}_{3\times4}$

$$A(BC) = \begin{bmatrix} 19+0+62 & 2+0+8 & 23+0+74 & 14+0+40 \\ -19+4+93 & -2-4+12 & -23+8+111 & -14+20+60 \\ 95+8+31 & 10-8+4 & 115+16+37 & 70+40+20 \end{bmatrix}$$

$$\underline{A(BC)} = \begin{bmatrix} 81 & 10 & 97 & 54 \\ 78 & 6 & 96 & 66 \\ 134 & 6 & 168 & 130 \end{bmatrix}_{3\times4} \Rightarrow \underline{RHS}$$

Here LHS = RHS Hence proved

$$(AB)C = \begin{bmatrix} 81 & 10 & 97 & 54 \\ 78 & 6 & 96 & 66 \\ 134 & 6 & 168 & 130 \end{bmatrix} \rightarrow \underline{LHS}$$

Now: $\underset{3\times4}{BC} = \begin{bmatrix} 1+18 & -1+3 & 2+21 & 5+9 \\ 2+0 & -2+0 & 4+0 & 10+0 \\ 1+30 & -1+5 & 2+35 & 5+15 \end{bmatrix}$

$$BC = \begin{bmatrix} 19 & 2 & 23 & 14 \\ 2 & -2 & 4 & 10 \\ 31 & 4 & 37 & 20 \end{bmatrix}$$

Matrix

Ques:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \rightarrow$ Show that:- $A^3 - 23A - 40I = 0$

Sol^n:- $A^2 = AA = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+6+12 & 2+(-4)+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9+(-2)+1 \\ 4+6+4 & 8+(-4)+2 & 12+2+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 19+12+32 & 38+(-8)+16 & 57+4+8 \\ 1+36+32 & 2+(-24)+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

Now:- LHS:- $A^3 - 23A - 40I$

$$\Rightarrow \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 63-23-40 & 46-46-0 & 69-69-0 \\ 69-69-0 & -6+46-40 & 23-23-0 \\ 92-92-0 & 46-46-0 & 63-23-40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Matrix

Ex:- Solve:- $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Solⁿ:- $\Rightarrow \begin{bmatrix}\cos^2\theta & \cos\theta\cdot\sin\theta\\ -\cos\theta\cdot\sin\theta & \cos^2\theta\end{bmatrix} + \begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}\underline{\cos^2\theta+\sin^2\theta} & \cos\theta\cdot\sin\theta+(-\sin\theta\cdot\cos\theta)\\ -\cos\theta\cdot\sin\theta+\sin\theta\cdot\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I_2$$

Matrix

Ex:- if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ → find K if $A^2 = KA - 2I$

Sol^n:- LHS:- $A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 9+(-8) & -6+4 \\ 12+(-8) & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \text{LHS}$$

RHS ⇒ $KA - 2I \Rightarrow k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix} = \text{RHS}$$

Now:- LHS = RHS

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

∴ $a_{11} = b_{11} \Rightarrow 1 = 3k - 2$

$\Rightarrow 3k = 3 \Rightarrow k = 1$

$\Rightarrow -2 = -2k \Rightarrow k = 1$

$\Rightarrow 4 = 4k \Rightarrow k = 1$

$\Rightarrow -4 = -2k - 2 \Rightarrow -4 + 2 = -2k$

$= -2 = -2k$

$= k = 1$