Matrix

Multiplication of Matrices :-

$A = \begin{bmatrix} 2 & 1 & 7 \\ 5 & 3 & -1 \end{bmatrix}_{2\times 3}$, $B = \begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 0 \\ 7 & 4 & 2 \end{bmatrix}_{3\times 3}$

$\Rightarrow$ when no. of Col. of Matrix A = no. of Rows of B

$\boxed{3 = 3}$ ✓

So we can Multiply.

$$A \cdot B = \begin{bmatrix} 2 & 1 & 7 \\ 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 0 \\ 7 & 4 & 2 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} \underbrace{2\times3+1\times1+7\times7}_{C_{11}} & \underbrace{2\times1+1\times(-1)+7\times4}_{C_{12}} & \underbrace{2\times2+1\times0+7\times2}_{C_{13}} \\ \underbrace{5\times3+3\times1+(-1)\times7}_{C_{21}} & \underbrace{5\times1+3\times(-1)+(-1)\times4}_{C_{22}} & \underbrace{5\times2+3\times0+(-1)\times2}_{C_{23}} \end{bmatrix}_{2\times3} \end{array} \begin{array}{c} \to R_1 \\ \to R_2 \end{array}$$

order, AB = no. of Row of A × no. of Column of B

order $\Rightarrow$ 2×3

$$= \begin{bmatrix} 6+1+49 & 2-1+28 & 4+0+14 \\ 15+3-7 & 5-3-4 & 10+0-2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 56 & 29 & 18 \\ 11 & -2 & 8 \end{bmatrix}_{2\times 3} \quad \underline{\underline{A}}$$

✓ $\underline{AB \neq BA}$ ✓

Matrix

Multiplication of Matrices:-

Ex. → $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$, $B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 6 & 1 \end{bmatrix}_{2\times 3}$

AB Define ? → $2 = 2$ ✓ → $\boxed{\text{order} = 2\times 3}$ ✓

BA Define ? → $3 \neq 2$ φ

$B = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$

$BA = \begin{bmatrix} 1+16 & 2+28 \\ 3+0 & 6+0 \end{bmatrix} = \begin{bmatrix} 17 & 30 \\ 3 & 6 \end{bmatrix}$

it is clear that $\boxed{AB \neq BA}$

Ex:- $A = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}_{2\times 2}$, $B = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}_{2\times 2}$ $\Rightarrow A\cdot B = \begin{bmatrix} 1+6 & 4+0 \\ 4+21 & 16+0 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 25 & 16 \end{bmatrix} = \underline{AB}$

AB → $\boxed{2=2}$ ✓ → order = $\underline{2\times 2}$ ✓

BA → $\boxed{2=2}$ ✓ → order = 2×2 ✓

But we can't $\underline{AB = BA}$

Matrix

Multiplication of Matrices:-

Ex.- $A = \begin{bmatrix} 2 & 1 \\ 5 & 7 \\ 3 & 0 \end{bmatrix}_{3\times 2}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \end{bmatrix}_{2\times 3}$

$a \cdot b = 0$

either → $\boxed{a = 0}$ or $\boxed{b = 0}$

$\boxed{A \cdot B = 0}$

either $\boxed{A = 0}$ ✓ $\boxed{B = 0}$ ✓

$A \cdot B \rightarrow \boxed{2 = 2}$ ✓ → $\boxed{\text{order} = 3\times 3}$

$B \cdot A \rightarrow \boxed{3 = 3}$ ✓ → $\boxed{\text{order} = 2\times 2}$ ✓

$AB \neq BA$

→ if order of A is $m\times n$ & order of B is $k\times l$ then Both AB & BA is define if:-

$AB \Rightarrow \boxed{n = k \ \& \ l = m}$

Matrix

Multiplication of Matrices :-

Ex. - $A = \begin{bmatrix} 0 & 1 \\ 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 0 & 0 \end{bmatrix}$

$\rightarrow AB = \begin{bmatrix} 0\times4+1\times0 & 0\times3+1\times0 \\ 0\times4+5\times0 & 0\times3+5\times0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\boxed{AB = 0}$ ✓

$a \cdot b = 0$

either $\rightarrow$ $a = 0$ or $b = 0$

✓ $\boxed{A \cdot B = 0}$

either $\boxed{A = 0}$ ✓ $\boxed{B = 0}$ ✓

if $A \cdot B = 0$ it does not mean either $A = 0$ or $B = 0$

Matrix

Multiplication of Matrices :-

Properties :-

i) Associative law :- if three Matrix A, B & C.

$$\therefore \boxed{(AB)C = A.(B.C)}$$

ii) distributive law :- i) $A(B+C) = AB + AC$

ii) $(A+B)C = AC + BC$

iii) Multiplicative Identity :- A → Square matrix, I → Identity Matrix

$$\boxed{AI = IA = A}$$

Hence proved

$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $I = \begin{bmatrix} 1 & \\ 0 & 1 \end{bmatrix}$

$AI = \begin{bmatrix} 2+0 & 0+1 \\ 0+0 & 0+3 \end{bmatrix}$

$AI = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = A$

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$IA = \begin{bmatrix} 2+0 & 1+0 \\ 0+0 & 0+3 \end{bmatrix}$

$IA = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = A$