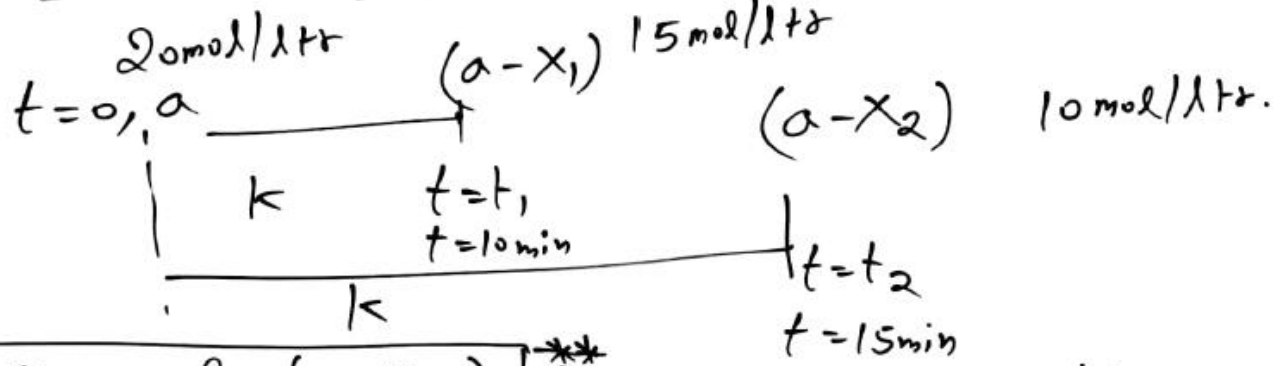
Interval Formula of Ist order Reaction



$$k = \frac{2.303}{t_2 - t_1} \log\left(\frac{a - x_1}{a - x_2}\right)$$ ***

Here :- t_1 & t_2 ⇒ time

$(a - x_1)$ ⇒ Conc. at time t_1

$(a - x_2)$ ⇒ -11- at time t_2

k ⇒ rate constant of Ist order rexn. (Sec^{-1}/min^{-1}/Hr^{-1})

General integrated rate equation (n^{th} order rate eq.) :-

$$kt = \frac{1}{(n-1)}\left[\frac{1}{(a-x)^{n-1}} - \frac{1}{a^{n-1}}\right] \quad ; \quad n \neq 1$$

Here $\Rightarrow$ n is order of rexn.; $t \rightarrow$ time; $a \rightarrow$ initial Conc.

$k \rightarrow$ rate constant ; $(a-x) \rightarrow$ conc. at time t.

Half life of n^{th} order Rexn :- $t \rightarrow t_{1/2}$

$x = a/2$; $a - x = a - a/2 = a/2$

Put in given eq.

$$kt_{1/2} = \frac{1}{(n-1)}\left[\frac{1}{(a/2)^{n-1}} - \frac{1}{a^{n-1}}\right]$$

$$k\, t_{1/2} = \frac{1}{(n-1)}\left[\frac{2^{n-1}}{a^{n-1}} - \frac{1}{a^{n-1}}\right]$$

$$k\, t_{1/2} = \frac{1}{(n-1)}\left[\frac{2^{n-1}-1}{a^{n-1}}\right] \quad (***)$$

$t_{1/2} \Rightarrow$ Half life.

$t_{1/2} \propto \frac{1}{a^{n-1}}$

If $a \to a_1$, $t_{1/2} \to (t_{1/2})_1$

$a \to a_2$, $t_{1/2} \to (t_{1/2})_2$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1} \quad (***)$$

IInd order Reaction :-

① Integrated rate eq. :- By using n^{th} order Rexn integrated rate eq.

$$kt = \frac{1}{(n-1)}\left[\frac{1}{(a-x)^{n-1}} - \frac{1}{a^{n-1}}\right] \; ; \; n \neq 1$$

For $n = 2$ [second order Rexn]

$$kt = \frac{1}{2-1}\left[\frac{1}{(a-x)^{2-1}} - \frac{1}{a^{2-1}}\right]$$

$$kt = 1\left[\frac{1}{(a-x)} - \frac{1}{a}\right] \Rightarrow kt = \left[\frac{a-(a-x)}{a\cdot(a-x)}\right]$$

$$kt = \frac{a - a + x}{a\cdot(a-x)} \Rightarrow \boxed{kt = \frac{x}{a\cdot(a-x)}}$$

$\rightarrow$ IInd order Rexn.

$k \rightarrow$ rate Const. of IInd order Rexn.

$t \rightarrow$ time

$a \rightarrow$ initial conc.

Here $\rightarrow x \rightarrow$ dissociated Conc. at time t

$(a-x) \rightarrow$ conc. at time t

Half life of IInd order Rexn :-

$t \rightarrow t_{1/2}$, $x = a/2$; $a - x = a - \frac{a}{2} = a/2$

Put in IInd order integrated rate eq. ***

$$k\, t_{1/2} = \frac{a/2}{a \cdot \left(\frac{a}{2}\right)} \Rightarrow \boxed{t_{1/2} = \frac{1}{a \cdot k}}$$

Half life of IInd order Rexn. is inversely proportional to initial Conc.

Q.1 90% of Ist order rexn was completed in 10 Hr. When will 99.9% reaction Complete.

Sol. $k_{90\%} = k_{99.9\%}$

$$\frac{2.303}{10}\log\left(\frac{100}{100-90}\right) = \frac{2.303}{t}\log\left(\frac{100}{100-99.9}\right)$$

$$\frac{1}{10}\log\left(\frac{100}{10}\right) = \frac{1}{t}\log\left(\frac{1000}{01}\right)$$

$$\frac{1}{10}\cdot 1 = \frac{1}{t}\log(10)^3 \Rightarrow \frac{1}{10} = \frac{3}{t}\log 10$$

$$t = 3\cdot 10\cdot 1 \Rightarrow \boxed{t = 30\,hr}\ \text{Ans.}$$

(2.2) 20% of I st order reaction was completed in 5 min. When will 60% of reaction complete.?

Sol. $k_{20\%} = k_{60\%}$

$$\frac{2.303}{5}\log\left(\frac{100}{100-20}\right) = \frac{2.303}{t}\log\left(\frac{100}{100-60}\right)$$

$$\frac{1}{5}\cdot\log\left(\frac{10\cancel{0}}{8\cancel{0}}\right) = \frac{1}{t}\log\left(\frac{10\cancel{0}}{4\cancel{0}}\right)$$

$$\frac{1}{5}\left[\log 10 - \log(2)^3\right] = \frac{1}{t}\left[\log 10 - \log(2)^2\right]$$

$$\frac{1}{5}\left[1 - 3\times 0.30\right] = \frac{1}{t}\left[1 - 2\times 0.30\right]$$

$$\frac{1}{5}\left[1 - 0.90\right] = \frac{1}{t}\left[1 - 0.60\right] \Rightarrow \frac{1}{5}\times 0.10 = \frac{1}{t}\times 0.4.$$

$$t = 5\times 4 \Rightarrow \boxed{t = 20\ \text{min}}\ \text{Ans.}$$

① $\log(m\cdot n) = \log m + \log n$

② $\log\left(\frac{m}{n}\right) = \log m - \log n$

③ $\log m^n = n\log m$

$\log 2 = 0.30$

$\log 3 = 0.48$

$\log 5 = 0.70$

$\log 7 = 0.84$

Q.3 For I^{st} order rexn Half life is 6 min. Cal. the rate Constant of I^{st} order rexn.

Sol. $t_{1/2} = 6\,min$, $k = ?$

$t_{1/2} = \frac{0.693}{k} \Rightarrow k = \frac{0.693}{6}$

$\boxed{k = 0.1155\,min^{-1}}$

Q.4. A I^{st} order reaction was 90% Complete in 40min. Cal. Half life of the reaction.

Sol. given $t_{90\%} = 40min.$, $t_{1/2} = ?$

$k = \frac{2.303}{t}\log\left(\frac{a}{a-x}\right)$

$k = \frac{2.303}{40}\log\left(\frac{100}{100-90}\right)$

$k = \frac{2.303}{40}\log\left(\frac{10}{1}\right) \Rightarrow \boxed{k = \frac{2.303}{40}}$

$t_{1/2} = \frac{0.693}{k} \Rightarrow t_{1/2} = \frac{2.303\log 2}{2.303} \times 40$

$t_{1/2} = 0.30 \times 40$

$= 3 \times 4$

$\boxed{t_{1/2} = 12\,min}$

Q.5. If the initial Conc. of reactants are double then Half life becomes Half. Cal. order of reaction.

Sol. let; $a_1 \longrightarrow a$; $(t_{1/2})_1 \longrightarrow (t_{1/2})$

$a_2 \longrightarrow 2a$; $(t_{1/2})_2 \longrightarrow \dfrac{(t_{1/2})}{2}$

$$\boxed{\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1}}$$

$$\frac{t_{1/2}}{\frac{t_{1/2}}{2}} = \left(\frac{2a}{a}\right)^{n-1}$$

$$2 = 2^{n-1} \Rightarrow 1 = n-1$$

$$\boxed{n = 1+1 = 2}$$

IInd order //